Name of college - S.S. college, J-Bad.
Deptt - Mathematics
Topic - Inverse of a Matrix
Time - 1.00 P.M to 1.45 P.M
class - B.Sc I (past Sub)
Date - 23-09-2020
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Study Material :→ A Square matrix A is invertible if there exists a square Matrix B of the same order such that

$$AB = BA = I \quad - (1)$$

Where I is the ~~unit~~ identity Matrix then B is defined as the inverse of the Matrix A.

Inverse of Matrix is denoted by A^{-1}

We have the result

$$A \cdot \operatorname{adj} A = \operatorname{adj} A \cdot A = |A| I$$

$$\Rightarrow A \frac{\operatorname{adj} A}{|A|} = \frac{\operatorname{adj} A}{|A|} \cdot A = I \quad - (11)$$

On Comparing (1) and (11) we get

$$B = \frac{\operatorname{adj} A}{|A|} = A^{-1}$$

= Inverse of A = Reciprocal of A

Show that, Every invertible Matrix possesses a unique inverse —

Proof :- Let A be a Square Matrix of order n

Also $|A| \neq 0$ i.e Matrix A is non Singular

Let Matrix A be invertible. Matrix.

Hence there Exists a Matrix B Such that -

$$AB = BA = I \quad \text{——— (i)}$$

If possible, let B_1 and B_2 be two inverses of A where $B_1 \neq B_2$

then we have

$$AB_1 = B_1A = I \quad \text{——— (ii)}$$

$$AB_2 = B_2A = I \quad \text{——— (iii)}$$

then $B_1 = B_1 I$

$= B_1(AB_2)$

$= (B_1A) B_2$

$= I B_2$

$= B_2$

$\therefore B_1 = B_2$

$\Rightarrow$ Hence our assumption is wrong.

therefore invertible matrix possesses a unique inverse.

for the Existence of the inverse of a square Matrix is that it must be Non Singular

$$\left[\begin{array}{l} A = [a_{ij}] \text{ be a square Matrix} \\ \text{then } |A| = 0 \Rightarrow \text{Matrix is Singular} \\ \qquad\quad \neq 0 \Rightarrow \text{Matrix is Non Singular} \end{array}\right]$$

Necessary Condition :→

Let A be a square matrix of order n and let B be the inverse of A. then By definition of inverse Matrix

$$AB = BA = I_n$$

$$\Rightarrow |AB| = |I| = 1$$

$$|A| \cdot |B| = 1$$

this equality is possible only when neither $|A|$ nor $|B|$ is zero.

$$\Rightarrow |A| \neq 0$$

$\Rightarrow A$ is Non-singular Matrix

Sufficient Condition :→ Let A be Non-Singular Matrix of order n

i.e $|A| \neq 0$

then, it is possible to define a Matrix B such that

$$B = \frac{\text{adj } A}{|A|}$$

Then $AB = A \dfrac{\operatorname{adj} A}{|A|} = \dfrac{A \operatorname{adj} A}{|A|}$

$$= \frac{|A| I}{|A|} = I$$

Similarly $BA = \dfrac{\operatorname{adj} A}{|A|} A$

$$= \frac{(\operatorname{adj} A) A}{|A|} = \frac{|A| I}{|A|} = I$$

Clearly $AB = BA = I$

Thus condition that A is invertible and B is the inverse of A and we have

$$B = \frac{\operatorname{adj} A}{|A|}$$

Theorem: If A, B be two non singular matrices of order n, then AB is also non-singular and $(AB)^{-1} = B^{-1} A^{-1}$

This Rule is known as "Reversal Law of inverses"

Proof. — Let A and B be two square matrix of order n

From the property of determinant —

$$|AB| = |A||B| \quad \text{——— (I)}$$

Since A and B are Non Singular Matrix

$\Rightarrow |A| \neq 0$

$|B| \neq 0$

therefore $|AB| \neq 0$

$\Rightarrow AB$ is Non-Singular Matrix

$\Rightarrow AB$ is invertible

Since $|A| \neq 0 \Rightarrow A^{-1}$ exists and

$$AA^{-1} = A^{-1}A = I \quad \text{——— (II)}$$

Again $|B| \neq 0 \Rightarrow B^{-1}$ exists. and

$$BB^{-1} = B^{-1}B = I \quad \text{——— (III)}$$

Also we have. $AI = IA = A$. ——— (IV)

$BI = IB = B$ ——— (V)

Let us define a Matrix C by setting/rule

$$C = B^{-1}A^{-1} \quad \text{——— (VI)}$$

then $C(AB) = (B^{-1}A^{-1})(AB$

$= B^{-1}(A^{-1}A)B$

$= B^{-1}(IB)$

$= B^{-1}B$

$= I$

Also $(AB)C = (AB)(B^{-1}A^{-1})$

$= A(BB^{-1})A^{-1}$

$= (AI)A^{-1}$

$= AA^{-1}$

$= I$

thus we get

$C(AB) = (AB)C = I$

it follows $C = B^{-1}A^{-1}$ is the inverse of AB

thus ~~$B^{-1}A^{-1}$~~ $(AB)^{-1} = B^{-1}A^{-1}$

thus we can say that the inverse of the product of two non-singular matrices is equal to the product of their inverses in reverse order.